

How to Calculate Cadence from Speed.

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This simple calculation frightens many people. It should not. There are a wealth of online based gear to speed calculators out there, but they serve as mysterious black boxes and enlighten no one. Here is the explanation how it works:

Simply put, it is your speed (in distance/time) divided by your development (distance/pedal revolution). This division yields revolutions per time, which is what RPM is.

$$\frac{\text{speed}}{\text{development}} = \frac{\text{dist./time}}{\text{dist./rev.}} = \frac{\text{dist.}}{\text{time}} \times \frac{\text{rev.}}{\text{dist.}} = \frac{\text{rev}}{\text{time}}$$

Development is how far you go with each pedal revolution. Your *development* is simply your *gear ratio* times your wheel *circumference*. Your *gear ratio* is your chainring's teeth divided by your cog's teeth, this is a unitless number. Your wheel *circumference* is your wheel *diameter* times *pi*. *Pi* is an unlegislatable number that is the ratio of a circle's circumference to its diameter, it is 3.14159 out to 5 digits and it goes infinitely. Conveniently 3.14159 rhymes with the phrase "secant, cosine, tangent, sine", which is a handy mnemonic.

Here is where most people get messed up. If your wheel diameter is in inches, your development is in inches. Your speedometer usually reads in miles per hour. In order for the simple speed/development equation to work, the distance measurements must be in the same units. In order for revolutions per time to come out into rpm (revolutions per minute) you must convert hours to minutes. These are all small simple calculations, but many people fail math and physics class because they panic when units need to change and just start multiplying away. Heck, one of the mars orbiters crashed because the design team and mission control were not on the same page unit-wise.

Here is what the equation looks like in units, rather than dimensions:

$$\frac{\text{speed}}{\text{development}} = \frac{\text{miles/hour}}{\text{inches/rev.}} = \frac{\text{miles}}{\text{hour}} \times \frac{\text{rev.}}{\text{inches}}$$

Now lets through in a few unit conversion terms to make it all kosher. There are 5280 feet per mile, 12 inches per foot, and 60 minutes per hour. If I express each of these as a ratio, they equal one and I can multiply them to the last term in the equation:

$$Y \left[\frac{\text{miles}}{\text{hour}} \times \frac{\text{rev.}}{\text{inches}} \right] \times 5280 \left[\frac{\text{feet}}{\text{mile}} \right] \times 12 \left[\frac{\text{inches}}{\text{foot}} \right] \times \frac{1}{60} \left[\frac{\text{hour}}{\text{min}} \right] = Y \times 1056 \left[\frac{\text{rev}}{\text{min.}} \right]$$

Y is your speed in MPH divided by your development in inches (units in brackets). Take that term, go through the unit conversions and you get your original Y number times 1056 equals your revolutions per minute. Hey! That's RPM!

Let us use the following word problem to demonstrate the calculation and the unit conversion:

Running the canonical 42x16 fixed gear in the mean streets of NYC, Bikey Becky is skitching a ride on an aggressive cabby who has green as far as he can see heading downtown on 2nd. She peeks over the cabby's shoulder and sees the speedometer creep up to 52 MPH before a tourist from jersey blows a red light on a cross street, causing the cabby to brake and Becky to skip stop just long enough to clear the back bumper of Jersey and shoot through the intersection unscathed. As she cruises down to alphabet city to a bar to meet some buddies for drinks, she looks down at her scarred up '73 Paramount track bike and wonders why her hips hurt so much. How many RPM's was Becky turning before she skip stopped?

Your first task with any word problem is to figure out what the question is, your second is to figure out what is needed information and ignore the rest. If you are sitting there wondering what skitching is or trying to remember if you can really head down town on second avenue in NYC, or if you are wondering if Bikey Becky is "hot" and if you can get her phone number, you FAIL. While interesting questions all, they do not help you solve the problem. The central issue is how many pedal revolutions per minute was Bikey Becky churning. The facts you need are that she is going 52 MPH in a 42-16 gear on a 700c bike (not explicitly stated, but an old Schwinn Paramount almost undoubtedly has 700c wheels, another common word problem trick, give you unnecessary info up front, but make you infer some necessary stuff.). So let's go back and restate the problem simply:

Rider is going 52 MPH in a 42-16 gear with a 700c wheel, how many RPM's?

Hey! That is much better. No visions of Bikey Becky pulling you on rides through Central Park and out to Coney Island, no wondering where one could meet her, just the facts. Now you can get out your thinky muscles and get to work. Speed is explicitly stated as 52 MPH, so now all we have left is to calculate development, convert some units and divide.

First figure the gear ratio: $42/16 = 2.625$. This means for every pedal revolution, Becky's rear wheel turns 2.625 times. The wheel diameter is about 27" as most 700c wheels are, so the circumference is: $27 * \pi = 84.8$ ". Thus the development is $2.625 * 84.8 = 222.6$ ".

Since we did the calculation above, we can simply divide 52 by 222.6 and then multiply by the correcting factor of 1056 to get RPM. So:
 $(52/222.6) * 1056 = 246$ RPM.

Man! No wonder Bikey Becky's hips are sore, that is some churning!

Now for kicks, lettuce (nyuk) go back a bit and do some weirdy unit play so that we can become facile at this both at home and in dangerously scary foreign locals, like Canada.

Imagine I told you that Becky saw that the cab was flying at 84 kilometers per hour (roughly 52 MPH, but in scary metric terms). We can go through and do the following:

Speed = 84 kph

Wheel diameter = 700 mm

Wheel Circumference = $700 * \pi = 2199\text{mm}$

Development = gear ratio * circumference = $2.625 * 2199 = 5772\text{mm} = 5.77\text{m} = 0.0058\text{km}$

So since we have the distances in kilometers, we can divide Speed by development and get revolutions per hour:

$84 / 0.0058 = 14482 \text{ revs/hour}$

Dividing by 60 we get revs per minute

$14482 / 60 = 241\text{rpm}$.

The numbers are not exactly the same, as 700mm is a rough metric approximation, while 27" is a rough English approximation. They differ by 15mm or so. However, it is useful to note that the cadence is only a few rpm off with 15mm change in tire diameter. This is akin to going from a 700x23 to a 700x30. This should tell you that being really exact does not buy you too much in this calculation.

It is also very useful to note that in a 2.625" gear ratio (70" gear for those who like gear inches) that cadence is a bit higher than 3 times the speed in kilometers per hour. This circumvents lots of figuring and can give you some idea of how fast your legs churned on that last downhill. If you are running a 68" gear, or a 2.5ish gear ratio, cadence is darn near exactly 3 times the speed in KPH.

Anyhoo, the long and short of it all is that it is merely simple unit analysis, which kills most people in math. Get past this and you can calculate your rpm from your speed in your head as you ride. Really...

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This is meant to help clarify the gear inches and cadence to RPM conversion and help with the Excel spreadsheet I made at <http://www.tariksaleh.com/bike/gears.html> .